Photogrammetric Space Resection

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Overview

- Introduction
- Problem Definition
- Importance of Space Resection
- Photogrammetry versus Computer Vision
- Mathematical Indirect Solutions
- Direct Solutions
- Comparison
- Conclusions
Photogrammetry is defined as a measurement technique where the coordinates of the points in 3D of an object are calculated after measurements made on 2D photographic images taken by metric camera.

The position and attitude of the camera (camera exterior orientation elements) during the exposure is an important factor in determining the required ground coordinates.

The process of determining the position and attitude of the camera is called Space Resection (SR).
Exterior Orientation Elements

The position of the camera during exposure is given in a three dimensional coordinate system as:

X, Y and Z

The attitude of the camera is defined as the three rotations $\omega$, $\Phi$ and $K$ about the X, Y and Z axes respectively to fit a given three dimensional coordinate system.
Space Resection

- The diagram below shows the SR problem
Importance of SR in Photogrammetry

- Importance:
- The problem is important for both photogrammetry and computer vision disciplines.
- Some of the photogrammetric applications of the space resection are:
  - Fixing ground coordinates by intersection from single photo after solving the space resection problem.
  - Photo Triangulation, using multiple photos (Bundle Adjustment)
  - Camera calibration (Tsai, 1987)
  - Head-Mounted Tracking System Positioning (Azuma and Ward, 1991)
  - Object Recognition
  - Ortho photo rectification
Importance of SR in Computer Vision

- SR applications in computer vision include:

- Head-Mounted Tracking System Positioning (Azuma and Ward, 1991)

- Robot picking and Robot navigation (Linnainmaa et al. 1988)

- Visual surveying in 3D input devices
- Head pose computation
Space Resection:
Photogrammetry-Computer Vision

- Space resection is therefore dealt with in both photogrammetry & computer vision.
- Here is a comparison between the two treatments.
- Difference is shown in terminology and in processing:
- Terms:
- Photogrammetry  V  Computer Vision
Photogrammetry V Computer Vision
Terms, coordinates
Photogrammetry V Computer Vision
Solution Concepts

Non-linear problem

Iterative Solution

No need for initial approximations

Collinearity Condition

Closed Form
Approximate Solutions

Here are some approximate solutions of the SR problem:

1- The Direct Linear Transformation (DLT), which is a method frequently used in photogrammetry and remote sensing.

2- The Church method proposed as a solution for single image resection (Slama, 1980).

3- A simplified absolute orientation method based on object-distances and vertical lines used when no control points are available. This method is largely applied in archaeology and architecture by non-photogrammetrists due to its simplicity.
Approximate Solutions

Continued:

4- A method of 3D conformal coordinate transformations (Dewitt, 1996) where a special formulation of the rotation matrix as a function of the azimuth and tilt is proposed.

5- An approximate solution of the spatial transformation (Kraus, 1997) which is particularly suitable when incomplete control points are used.
Church’s solution (1945)

First iterative solution was published by Church, 1945, 1948.

This is done by linearizing collinearity equations, (Wolf, 1980; Salama, 1980)

It needs a good starting value which constitutes an approximate solution.

Approximate initial values which can be known to 10% accuracy for scale and distances and to within 15° for rotation angles would then be adjusted by the solution.

See Seedahmed, 2008 for autonomous initial values for exterior orientation parameters (EOP)
Collinearity Condition

The exposure station of a photograph (L), an object point (A) and its photo image (a) all lie along a straight line (L, a, A).
Image & Ground Coordinate Systems

• Ground Coordinate System - X, Y, Z

In Ground Coordinate System
• Exposure Station Coordinates  \( L( X_L, Y_L, Z_L) \)
• Object Point (A) Coordinates  \( A( X_a, Y_a, Z_a) \)
• Image coordinate system \((x', y', z')\) parallel to ground coordinate system \((XYZ)\)

In image Coordinate System
• image point (a) coordinates  \( a(x'_a, y'_a, z'_a) \)
• \( x'_a, y'_a \) and \( z'_a \) are related to the measured photo coordinates \( x_a, y_a \), focal length \((f)\) and the three rotation angles omega, phi and kappa.
Developed in a sequence of three independent two-dimensional rotations.

- **ω rotation about x’ axis**
  \[ x_1 = x' \]
  \[ y_1 = y'\cos \omega + z'\sin \omega \]
  \[ z_1 = -y'\sin \omega + z'\cos \omega \]

- **f rotation about y’ axis**
  \[ x_2 = -z_1\sin f + x_1\cos f \]
  \[ y_2 = y_1 \]
  \[ z_2 = z_1\cos f + x_1\sin f \]

- **κ rotation about z’ axis**
  \[ x = x_2\cos \kappa + y_2\sin \kappa \]
  \[ y = -x_2\sin \kappa + y_2\cos \kappa \]
  \[ z = z_2 \]
### Rotation Matrix

The sum of the squares of the three “direction cosines” in any row or in any column is unity.

\[
X = MX'
\]

\[
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix} =
\begin{bmatrix}
    m_{11} & m_{12} & m_{13} \\
    m_{21} & m_{22} & m_{23} \\
    m_{31} & m_{32} & m_{33}
\end{bmatrix}
\begin{bmatrix}
    x' \\
    y' \\
    z'
\end{bmatrix}
\]

\[
x = m_{11}x' + m_{12}y' + m_{13}z'
\]
\[
y = m_{21}x' + m_{22}y' + m_{23}z'
\]
\[
z = m_{31}x' + m_{32}y' + m_{33}z'
\]
Collinearity Condition Equations from Similar Triangles

Collinearity condition equations developed from similar triangles (Wolf)

\[
\frac{x'_a}{X_a - X_L} = \frac{y'_a}{Y_a - Y_L} = \frac{-z'_a}{Z_a - Z_L}
\]

\[
x_a = m_{11} \left( \frac{X_a - X_L}{Z_a - Z_L} \right) z'_a + m_{12} \left( \frac{Y_a - Y_L}{Z_a - Z_L} \right) z'_a + m_{13} \left( \frac{Z_a - Z_L}{Z_a - Z_L} \right) z'_a
\]

\[
y_a = m_{21} \left( \frac{X_a - X_L}{Z_a - Z_L} \right) z'_a + m_{22} \left( \frac{Y_a - Y_L}{Z_a - Z_L} \right) z'_a + m_{23} \left( \frac{Z_a - Z_L}{Z_a - Z_L} \right) z'_a
\]

\[
z_a = m_{31} \left( \frac{X_a - X_L}{Z_a - Z_L} \right) z'_a + m_{32} \left( \frac{Y_a - Y_L}{Z_a - Z_L} \right) z'_a + m_{33} \left( \frac{Z_a - Z_L}{Z_a - Z_L} \right) z'_a
\]

* Dividing \( x_a \) and \( y_a \) by \( z_a \)

* Substitute \(-f\) for \( z_a \)

* Correcting the offset of Principal point \((x_o, y_o)\)

\[
x_a = x_o - f \left[ \frac{m_{11}(X_a - X_L) + m_{12}(Y_a - Y_L) + m_{13}(Z_a - Z_L)}{m_{31}(X_a - X_L) + m_{32}(Y_a - Y_L) + m_{33}(Z_a - Z_L)} \right]
\]

\[
y_a = y_o - f \left[ \frac{m_{21}(X_a - X_L) + m_{22}(Y_a - Y_L) + m_{23}(Z_a - Z_L)}{m_{31}(X_a - X_L) + m_{32}(Y_a - Y_L) + m_{33}(Z_a - Z_L)} \right]
\]
Collinearity Condition equations from Vector Triangle

- I prefer using this approach when teaching to allow students to understand the principle:

- \( XA = XO + \lambda xa \)
- \( XA = 3D \) object space coordinates
- \( XO = \) exposure station space coordinates
- \( \lambda = \) scale
- \( xa = \) image coordinates
Collinearity Equations

- Nonlinear
- Nine unknowns
  \( \omega, f, \kappa \)
  \( X_A, Y_A \) and \( Z_A \)
  \( X_L, Y_L \) and \( Z_L \)

Taylor’s Theorem is used to linearize the nonlinear equations

\[
\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n
\]

substituting

\[
q = m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)
\]

\[
r = m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)
\]

\[
s = m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)
\]
Rewriting the Collinearity Equations

\[ F = x_o - f \frac{r}{q} = x_a \]
\[ G = y_o - f \frac{s}{q} = y_a \]

Taylor’s Theorem

\[ F_0 + \left( \frac{\partial F}{\partial \omega} \right)_0 d\omega + \left( \frac{\partial F}{\partial \phi} \right)_0 d\phi + \left( \frac{\partial F}{\partial \kappa} \right)_0 d\kappa + \left( \frac{\partial F}{\partial X_L} \right)_0 dX_L + \left( \frac{\partial F}{\partial Y_L} \right)_0 dY_L + \left( \frac{\partial F}{\partial Z_L} \right)_0 dZ_L \]
\[ + \left( \frac{\partial F}{\partial X_A} \right)_0 dX_A + \left( \frac{\partial F}{\partial Y_A} \right)_0 dY_A + \left( \frac{\partial F}{\partial Z_A} \right)_0 dZ_A = x_a \]
\[ G_0 + \left( \frac{\partial G}{\partial \omega} \right)_0 d\omega + \left( \frac{\partial G}{\partial \phi} \right)_0 d\phi + \left( \frac{\partial G}{\partial \kappa} \right)_0 d\kappa + \left( \frac{\partial G}{\partial X_L} \right)_0 dX_L + \left( \frac{\partial G}{\partial Y_L} \right)_0 dY_L + \left( \frac{\partial G}{\partial Z_L} \right)_0 dZ_L \]
\[ + \left( \frac{\partial G}{\partial X_A} \right)_0 dX_A + \left( \frac{\partial G}{\partial Y_A} \right)_0 dY_A + \left( \frac{\partial G}{\partial Z_A} \right)_0 dZ_A = y_a \]

- \( F_0 \) and \( G_0 \) are functions \( F \) and \( G \) evaluated at the initial approximations for the nine unknowns
- \( d\omega, \ d\phi, \ dk \) are the unknown corrections to be applied to the initial approximations
- The rest of the terms are the partial derivatives of \( F \) and \( G \) wrt to their respective unknowns at the initial approximations
Applying LSM to Collinearity Equations

- Residual terms must be included in order to make the equations consistent

\[ b_{11} d\omega + b_{12} d\phi + b_{13} d\kappa - b_{14} dX_L - b_{15} dY_L - b_{16} dZ_L + b_{14} dX_A + b_{15} dY_A + b_{16} dZ_A = J + V_{xa} \]
\[ b_{21} d\omega + b_{22} d\phi + b_{23} d\kappa - b_{24} dX_L - b_{25} dY_L - b_{26} dZ_L + b_{24} dX_A + b_{25} dY_A + b_{26} dZ_A = K + V_{ya} \]

\[ J = x_a - F_o \quad ; \quad K = y_a - G_o \]

b terms are coefficients equal to the partial derivatives
Numerical values for these coefficient terms are obtained by using initial approximations for the unknowns.

The terms must be solved iteratively (computed corrections are added to the initial approximations to obtain revised approximations) until the magnitudes of corrections to initial approximations become negligible.
Formulate the collinearity equations for a number of control points whose X, Y and Z ground coordinates are known and whose images appear in the tilted photo. The equations are then solved for the six unknown elements of exterior orientation which appear in them.

Space Resection collinearity equations for a point A

\[
\begin{align*}
    b_{11} d\omega + b_{12} d\phi + b_{13} d\kappa - b_{14} dX_L - b_{15} dY_L - b_{16} dZ_L &= J + V_{xa} \\
    b_{21} d\omega + b_{22} d\phi + b_{23} d\kappa - b_{24} dX_L - b_{25} dY_L - b_{26} dZ_L &= K + V_{ya}
\end{align*}
\]

A two dimensional conformal coordinate transformation is used

\[
\begin{align*}
    X &= ax' - by' + T_x \\
    Y &= ay' + bx' + T_y
\end{align*}
\]

X, Y – ground control coordinates for the point

x’, y’ – ground coordinates from a vertical photograph

a, b, Tx, Ty – transformation parameters
This method does not require fiducial marks and can be solved without supplying initial approximations for the parameters.

Collinearity equations along with the correction for lens distortion:

\[ x_a - \delta_x = x_0 - f_x \left[ \frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} \right] \]

\[ y_a - \delta_y = x_0 - f_y \left[ \frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)} \right] \]

- \( d_x, d_y \) – lens distortion
- \( f_x \) – pd in the x direction
- \( f_y \) – pd in the y direction

Rearranging the above two equations:

\[ x_a - \delta_x = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1} \]

\[ y_a - \delta_y = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1} \]

\[ L_4 = \frac{(x_0 m_{31} - f_x m_{11})}{L} \]
\[ L_5 = \frac{(x_0 m_{32} - f_x m_{12})}{L} \]
\[ L_6 = \frac{(x_0 m_{33} - f_x m_{13})}{L} \]
\[ L_7 = \frac{(y_0 m_{31} - f_y m_{21})}{L} \]
\[ L_8 = \frac{(y_0 m_{32} - f_y m_{22})}{L} \]
\[ L_9 = \frac{(y_0 m_{33} - f_y m_{23})}{L} \]
The resulting equations are solved iteratively using LSM

\[
\begin{bmatrix}
X_L \\
Y_L \\
Z_L
\end{bmatrix} =
\begin{bmatrix}
L_1 & L_2 & L_3 \\
L_5 & L_6 & L_7 \\
L_9 & L_{10} & L_{11}
\end{bmatrix}^{-1}
\begin{bmatrix}
L_4 \\
L_8 \\
1
\end{bmatrix}
\]

- Advantages
  - No initial approximations are required for the unknowns.
- Limitations
  - Requirement of at least six 3D object space control points
  - Lower accuracy of the solution as compared with a rigorous bundle adjustment

\[
L_8 = y_0 + f_y (m_{21} X_L + m_{22} Y_L + m_{23} Z_L) / L \\
L_9 = m_{31} / L \\
L_{10} = m_{32} / L \\
L_{11} = m_{33} / L \\
L = -(m_{31} X_L + m_{32} Y_L + m_{33} Z_L)
\]
Since the indirect solution requires initial approximate values for the exterior orientation parameters, and in computer vision problems the approximate values are not known, a direct solution is a must.

Six approaches of direct solution were presented and tested by Haralick, et al, 1994.
Direct Retrieval of EOP using 2D Projective Transformation

- Seedahmed (2006) presented a direct algorithm to retrieve the EOPs from the 2D projective transformation, based on a direct relationship between the 2D projective transformation and the collinearity model using a normalization process. This leads to a direct matrix correspondence between the 2D projection parameters and the collinearity model parameters: hence, a direct matrix factorization to retrieve the EOPs.
Space resection in photogrammetry using collinearity condition without linearisation

Said Easa (2010) presented an optimization model for space resection with or without redundancy that requires no linearization, iterations, or initial approximate values.

The model, which is nonlinear and nonconvex, is solved using advanced Excel-based optimization software that has been recently developed.

The proposed model is simple and converges to the global optimal solution very quickly.
The classical iterative method based on Euler angle using collinearity equations usually fail because the initial values are poor or unknown.

For this reason, H. Zeng, (2010) used Rodrigues matrix to represent the rotation matrix, and then established the mathematical model of space resection based on Rodrigues matrix, finally, he presented the solution of the model. This algorithm need linearization and iterative process, but has no initial values problem, regardless of the size of the Euler angle, and is fast and effective.
Newton-Raphson Search Solution
(no initial values)

- Said Easa (2007) described the geometry of the 3-point resection and the difficulties with N-R method.
- He presented a new Excel-based method that identifies the four solutions of the quartic polynomial.
- The method does not need initial estimates of the roots.
Explicit Solution with LSA to Redundant Control

- Smith 2006 suggested discrimination of the four resulting solutions by using redundant control and applying least squares adjustment (LSA) for the general case where photography is not nearly vertical.

- All four solutions will be examined in the light of the available redundant control.

- The disadvantage of the method is the need for redundant control.
Wang et al proposed a model based approach to space resection. The method recovers a camera’s location and orientation relative to an object coordinate system up to a scale factor without the use of any GCP or vanishing point.

The mathematical basis for this approach is the equivalence between the vector normal to the interpretation plane in the image space and the vector normal to the rotated interpretation plane in the object space.

A two-step iterative scheme for recovering camera orientation that, unlike existing methods, does not require a good initial guess for the rotation.
Non-Rigid Approach, continued

- Instead, the good initial estimate for the rotation is computed directly by using coplanarity constraints. A non-linear least squares minimization procedure is then applied to determine camera orientation accurately.

- The camera translation and predefined model parameters are determined based on the calculated rotation through a linear least squares minimization.

- Unlike existing methods, this method does not require a model-to-image fitting process, and is more effective and faster than previous approaches.
Non-Rigid Solution

- Object line 6-7 & image line 6-7
Straight lines intersection approach

- Lagunes & Battle (2009) proposed a technique based on determining the coordinates of the exposure station by **intersection of the straight lines through the three known ground control points.**
- The required azimuths of these lines are obtained from the geometric relationships between two similar triangles.
- Numerical solutions that show the good performance and accuracy of the solution were reported.
Recursive Straight Lines Solution

- Tomaselli and Tozzi (1996) developed a space resection solution using an explicit math model relating straight lines as features, applying Kalman Filtering.

- An iterative process using sequential estimated camera location parameters to feed back to the feature extraction leading to a gradual reduction of image space for feature searching.

- Results show highly accurate space resection parameters are obtained as well as a progressive processing time reduction.
Urban and Stroner (2012), Elnima (2013) suggested an optimization model based on genetic evolution algorithm to solve the three point space resection problem. The solution does not need linearization nor redundancy. The proposed model is simple and converges to the global optimal solution. The disadvantage is the high computational demand.
Some more Direct Solutions

Novel direct parametrization of prespective 3-point problem, by Kneip et.al

H. Zeng, 2012: Non-Iterative solution: camera distance equations and then camera pose solution
Applications

- Automobile Construction
- Machine Construction, Metalworking, Quality Control
- Mining Engineering
- Objects in Motion
- Shipbuilding
- Structures and Buildings
- Traffic Engineering
- Biostereometrics
Conclusions

- Collinearity condition and collinearity equations are more suitable for photogrammetric students who want to go deep in sight of the problem, considering single photo.
- Iteration solution needing initial approximations of parameters give good accuracy suitable for surveying needs. It makes no problem with the use of computers.
- Initial approximations can now be easily obtained either using simple DLT or when GPS is used.
Thank you..